CALCULATION OF EXPLOSIVE ACTION IN BRITTLE ROCK. CASE OF FRACTURE WITH FORMATION OF TEARING CRACKS

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The problem of explosive action in brittle rock in the general form was studied in [1]. Here we examine one case of this problem corresponding to relative low pressures in the explosive cavity, when fracture takes place only by the formation of radial cracks.

The solution in the region which has not yet fractured by radial cracks under the action of the explosion is described by the equations [1]

$$\begin{gathered}
\sigma_r = -\rho C_0^2\left\{\frac{f^{\cdot\cdot}(\zeta)}{x} + \frac{2(1-\sigma)}{1-\sigma}\left[\frac{f^{\cdot}(\zeta)}{x^2} + \frac{f(\zeta)}{x^3}\right]\right\} - P_h \\
\sigma_\varphi = \sigma_\theta = -\rho C_0^2\left\{\frac{\sigma}{1-\sigma}\frac{f^{\cdot\cdot}(\zeta)}{x} - \frac{1-2\sigma}{1-\sigma}\left[\frac{f^{\cdot}(\zeta)}{x^2} + \frac{f(\zeta)}{x^3}\right]\right\} - P_h \\
V = C_0\left[\frac{f^{\cdot\cdot}(\zeta)}{x} + \frac{f^{\cdot}(\zeta)}{x^2}\right], \quad u = r_0\left[\frac{f^{\cdot}(\zeta)}{x} + \frac{f(\zeta)}{x^2}\right] \\
\zeta = \tau - x, \ x = r / r_0, \ \tau = C_0 t / r_0
\end{gathered} \tag{1}$$

Here σ_r, σ_θ, σ_φ are the stresses on the coordinate planes which are by virtue of spherical symmetry the principal planes; V is the radial velocity of the particles; u is the displacement in the radial direction; x, τ are dimensionless coordinates; C_0 is the sound speed in the unfractured material; r_0 is the initial radius of the cavity; t is time; r is the Lagrangian coordinate; σ is the Poisson coefficient; ρ is the initial density. The single undefined function $f(\zeta)$ in these equations is found from the problem boundary conditions.

If the initial pressure P_0 in the cavity is small, fracture does not occur and the cavity will radiate an elastic wave described by (1), in which the function $f(\zeta)$ has the form

$$\begin{gathered}
f(\zeta) = a_1(p_0 - p_h)\left\{1 - a_2 e^{-a_3(\zeta+1)}\sin\left[a_4(\zeta+1) + \arcsin\frac{1}{a_2}\right]\right\} \\
a_1 = \frac{1-\sigma}{2(1-2\sigma)}, \quad a_2 = \sqrt{2(1-\sigma)}, \quad a_3 = \frac{1-2\sigma}{1-\sigma} \\
a_4 = \frac{\sqrt{1-2\sigma}}{1-\sigma}, \quad p_0 \equiv \frac{P_0}{\rho C_0^2}, \qquad p_h = \frac{P_h}{\rho C_0^2}
\end{gathered} \tag{2}$$

where P_h is the initial hydrostatic pressure in the medium.

We take the usual fracture conditions

$$\sigma_\theta = \sigma_* \qquad \text{(by tearing)} \tag{3}$$

$$\sigma_r - \sigma_\theta = -2\tau_* \qquad \text{(by shearing)} \tag{4}$$

where σ_*, τ_* are the strength constants of the material. Let us establish the initial conditions for which fracture begins by tearing. From (1) and (2) we have

$$\begin{gathered}
\sigma_r\Big|_{\substack{\tau=0\\x=1}} = -P_0, \quad \sigma_\theta\Big|_{\substack{\tau=0\\x=1}} = -\frac{\sigma}{1-\sigma}P_0 - \frac{1-2\sigma}{1-\sigma}P_h \\
(\sigma_r - \sigma_\theta)_{\substack{\tau=0\\x=1}} = -\frac{1-2\sigma}{1-\sigma}(P_0 - P_h)
\end{gathered} \tag{5}$$

Erevan. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 11, No. 3, pp. 89-97, May-June, 1970. Original article submitted November 14, 1969.

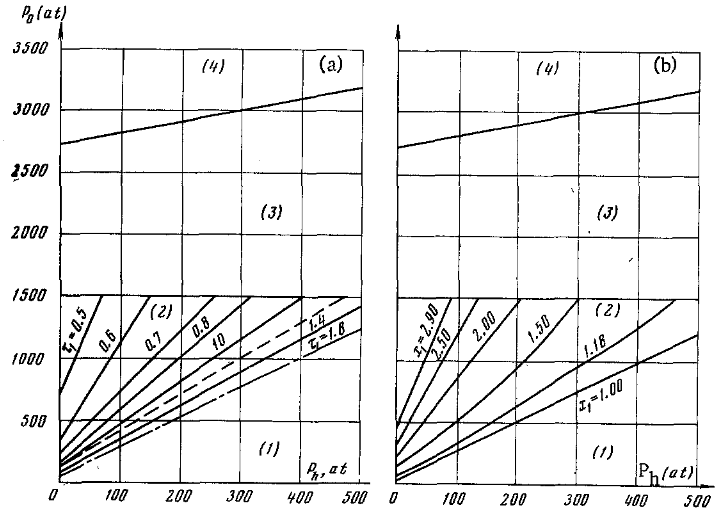

Fig. 1

Hence we see that at the initial moment the stresses σ_r and σ_θ are compressive and, consequently, fracture by tearing at the initial moment does not occur; nor does fracture take place by shearing if the condition is satisfied:

$$P_0 \leqslant P_h + \frac{2(1-\sigma)}{1-2\sigma}\tau_* \tag{6}$$

Let us examine the case in which the condition (6) is satisfied and let us find the nature of the motion which is described at the initial moment by (1) and (2).

The radial stress σ_r at the edge of the cavity is constant, while the ring stress σ_θ there will vary with time. From (1) and (2) it is easy to obtain the expressions for $\sigma_\theta(\tau, 1)$ and calculate the time τ_1 when the fracture condition (3) is reached at the cavity. The equation for τ_1 has the form

$$P_0 = P_h + \frac{2(\sigma_* + P_h)}{1 - 2a_1 e^{-a_3\tau_1}[a_5 \sin\psi_1 + a_6\cos\psi_1]}$$

$$a_5 = \frac{\sigma(1+\sigma)}{2(1-\sigma)^2}, \quad a_6 = \frac{\sigma + \sqrt{1-2\sigma}}{1-\sigma}, \quad \psi_1 = a_4\tau_1 + \arcsin\frac{1}{a_2} \tag{7}$$

In order to determine the final condition for which fracture will take place only by tearing, we find the value of P_0 for which both (3) and (4) are met for the point $x = 1$, i.e.,

$$P_0^* = 2\tau_* - \sigma_* \tag{8}$$

Thus, if $P_0 < P_0^*$ fracture is possible only by tearing, and it begins at the cavity surface at the time τ_1 defined by (7).

Figure 1a shows for granite ($\sigma_* = 45$ kg/cm^2, $\tau_* = 780$ kg/cm^2, $\sigma = 0.3$ Young's modulus $E = 2.22 \cdot 10^5$ kg/cm^2 [2]) the regions of P_0 and P_h variation in which fracture does not occur (region 1), fracture occurs by tearing (region 2), fracture occurs by shearing when the condition (4) is reached at the cavity (region 3), or fracture by shearing begins at the cavity immediately at the initial time (region 4).

In region 2 there are shown the (straight) isolines of $\tau_1(P_0, P_h)$ — the moment of onset of fracture by tearing at the cavity. The (dash-dot) line $\tau_1 = 1.8$ corresponds to the case in which the condition (3) arises at the cavity but propagation of the fracture front does not take place (boundary of regions 1 and 2). The dashed line is the analogous boundary for the static solution of the problem. This line is located above the dynamic loading line (dash-dot). Thus, under dynamic loading conditions the ability of the medium to withstand without fracture the pressure applied in the cavity is less than for static loading.

In the present study we examine only those initial conditions for which the point (P_0, P_h) falls in region 2. In this case, after the time τ_1 there will propagate from the cavity surface into the depth of the medium in accordance with the a priori unknown law $x = x_1(\tau)$ a spherical fracture front, spreading in the medium a system of radical normal-tearing-mode cracks. The solution in the region $x \geq x_1(\tau)$ is given as

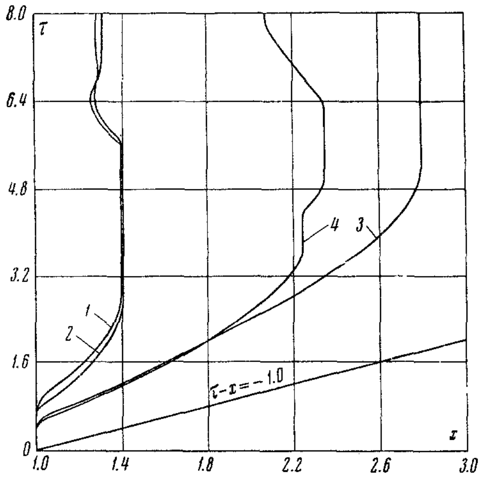

Fig. 2

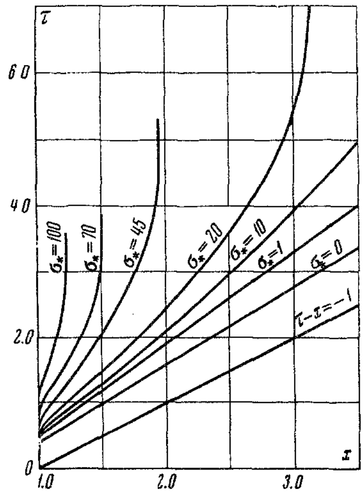

Fig. 3

before by (1), and the unknown function $f(\zeta)$ is found from the condition at the fracture front, i.e., with account for the solution in the region $1 \le x \le x_1(\tau)$.

In the fracture region the stress, velocity, and displacement fields are defined by the equations [1]

$$\begin{gathered}
\sigma_r = -\rho C_0^2 \lambda^2 \left[\frac{f_1^{\cdot}(\xi) - f_2^{\cdot}(\eta)}{x} + \frac{f_1(\xi) + f_2(\eta)}{x^2}\right] \\
\sigma_\varphi = \sigma_\theta \equiv 0, \quad V = \frac{C_0 \lambda}{x}\,[f_1^{\cdot}(\xi) + f_2^{\cdot}(\eta)] \\
u = \frac{r_0}{x}\left[f_1(\xi) + f_1(\eta) + \frac{1-\sigma}{1+\sigma} p_h x^2\right] \\
\xi = \lambda\tau - x, \qquad \eta = \lambda\tau + x \\
C_1 = \left(\frac{E}{\rho}\right)^{1/2}, \quad \lambda = \frac{C_1}{C_0} = \left[\frac{(1-2\sigma)(1+\sigma)}{1-\sigma}\right]^{1/2}
\end{gathered} \tag{9}$$

where C_1 is the speed of sound in the material fractured by radial cracks.

In (1) and (9) we have the three unknown functions $f(\zeta)$, $f_1(\xi)$, $f_2(\eta)$, which must be found from the condition at the cavity and from the joining condition at the fracture front, i.e., on the line $x = x(\tau)$, which is also to be determined. These conditions finally reduce to the system of relations [1]

$$\begin{gathered}
\frac{\sigma}{1-\sigma} f^{\cdot\cdot}(\zeta_1) - \frac{1-2\sigma}{1-\sigma}\left[\frac{f^{\cdot}(\zeta_1)}{x_1(\tau)} + \frac{f(\zeta_1)}{x_1^2(\tau)}\right] = -(\Sigma_* + p_h)\,x_1(\tau) \\
f^{\cdot}(\zeta_1) + \frac{f(\zeta_1)}{x_1(\tau)} = f_1(\xi_1) + f_2(\eta_1) + \frac{1-\sigma}{1+\sigma} p_h x_1^2(\tau) \\
f^{\cdot\cdot}(\zeta_1) + \frac{2(1-2\sigma)}{1-\sigma}\left[\frac{f^{\cdot}(\zeta_1)}{x_1(\tau)} + \frac{f(\zeta_1)}{x_1^2(\tau)}\right] - \lambda^2\left[f_2^{\cdot}(\xi_1) - f_2^{\cdot}(\eta_1) + \frac{f_1(\xi) + f_2(\eta_1)}{x_1(\tau)} + p_h x_1(\tau)\right] \\
= x_1^{\cdot}(\tau)\left\{f^{\cdot\cdot}(\zeta_1) + \frac{f^{\cdot}(\zeta_1)}{x_1(\tau)} + \lambda\,[f_1^{\cdot}(\xi_1) + f_2^{\cdot}(\eta_1)]\right\} \\
\lambda^2\,[f_1^{\cdot}(\xi^\circ) - f_2^{\cdot}(\eta^\circ) + f_1(\xi^\circ) + f_2(\eta^\circ)] = p_0\left[1 + f_1(\xi^\circ) + f_2(\eta^\circ) + \frac{1-\sigma}{1+\sigma} p_h\right]^{-3\gamma}, \\
\Sigma_* \equiv \frac{\sigma_*}{\rho C_0^2} \\
\zeta_1 = \tau - x_1(\tau), \qquad \xi_1 = \lambda\tau - x_1(\tau), \\
\eta_1 = \lambda\tau + x_1(\tau) \quad \xi^\circ = \lambda\tau - 1, \quad \eta^\circ = \lambda\tau + 1
\end{gathered} \tag{10}$$

where γ is the adiabatic exponent of the explosion products (for high pressures $\gamma \approx 3$, for low pressures $\gamma \approx 1.25$).

The explosion wave propagation process can be broken down into the following sequential stages:

1) in the course of the time $0 \le \tau \le \tau_1$ the cavity surface radiates an elastic wave and fracture does not occur in the medium;

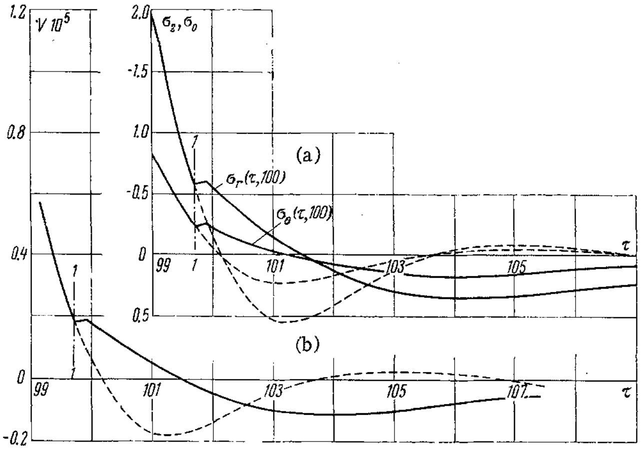

Fig. 4

2) an elastic wave propagates in the unfractured medium, the medium around the cavity fractures; the boundary between the unfractured and fractured zones (fracture shock wave) propagates from the cavity surface into the depth of the medium;

3) the fracture front stops, an elastic wave propagates through the unfractured medium, the boundary is a contact discontinuity; new fracture of the medium does not take place;

4) the contact discontinuity displaces into the fractured region; radiation of elastic waves continues.

The first stage is described completely by (1), (2). The second stage ($\tau_2 \le \tau \le \tau_2$) is described by (10) ($\tau_2$ is the time when the fracture front comes to a stop, i.e., when $\dot{x}_1(\tau_2) = 0$). The third stage ($\tau_2 \le \tau \le \tau_3$) is described by (10), in which the first equation must be replaced by the equation $x_1(\tau) = \text{const}$.

The solution obtained in this fashion will correspond to the absence of new fractures, and the stress σ_θ at the fracture boundary on the unfractured region side will decrease from the value σ_*. If at the moment $\tau = \tau_3$ the stress σ_θ at the fracture boundary on the unfractured material side vanishes the fourth stage arises, while if it approaches its positive asymptotic value the third stage will continue up to $\tau = \infty$. The fourth stage is described by equations obtained from (10) if in the third equation of this system the right side is equated to zero and we set $\Sigma_* = 0$. The resulting solution corresponds to the fact that as a result of the sign change of σ_* a compressive stress develops at the boundary of the fractured region and the boundary will displace into the fractured material, closing the cracks. Thereafter the boundary $x = x_1(\tau)$ may oscillate about the position which it approaches asymptotically. If in the process of these oscillations the front $x = x_1(\tau)$ reaches the true boundary of the unfractured region, then the solution must be constructed with account for the new fracture, i.e., from (10). The numerical solution method for this problem is described in [1].

Equation (10) breaks down into two systems of ordinary differential equations for which the Cauchy problem is posed, and these systems are integrated sequentially each time for the new Cauchy conditions and for the new interval of variation of the independent variable with use of the solution of the preceding system. For the initial conditions we must construct the asymptotic solution in the region $\tau_1 \le \tau \le \tau_1 + \Delta\tau$, where $\Delta\tau$ – the initial segment – is a given quantity. The existence of the asymptotic solution of (10) in $\Delta\tau$ – the vicinity of the initial point $(\tau_1, 1)$ – means that the function $f_1(\xi_1)$ will be known in the interval

$$\xi_{11} = \lambda\tau_1 - 1 \leqslant \xi_1 \leqslant \xi_{12} = \lambda(\tau_1 + \Delta\tau) - 1 \ .$$

Assuming the function $f_1(\xi_1)$ known in the interval $[\xi_{11}, \xi_{12})$, from the first three equations (10) we obtain the system for determining the functions $f(\xi_1)$, $f_2(\eta_1)$, and $x_1(\tau)$. Considering them as functions of the argument $\zeta_1 = \tau - x_1(\tau)$ and converting from differentiation with respect to the variables ξ_1, η_1, τ to differentiation with respect to ζ_1 with account for the conversion equations

$$\frac{d}{d\xi_1} = \frac{d}{d\zeta_1}\left[(\lambda - 1)\frac{d\tau}{d\zeta_1} + 1\right]^{-1}, \quad \frac{d}{d\eta_1} = \frac{d}{d\zeta_1}\left[(\lambda + 1)\frac{d\tau}{d\zeta_1} - 1\right]^{-1}$$
$$\frac{d}{d\tau} = \frac{d}{d\zeta_1}\left(\frac{d\tau}{d\zeta_1}\right)^{-1} \qquad (11)$$

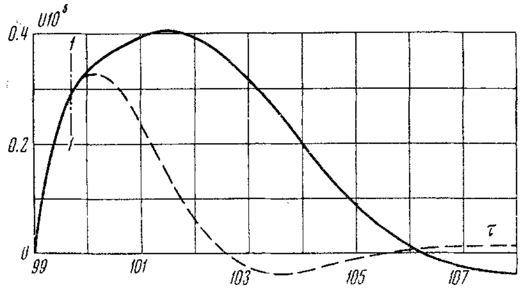

Fig. 5

we reduce the first three equations (10) to the following system of ordinary differential equations for finding the functions $f(\zeta_1)$, $f_2(\eta_1) = g_2(\zeta_1)$ and $\tau(\zeta_1)$:

$$\begin{gathered}
\frac{\sigma}{1-\sigma}\frac{f^{\bullet\bullet}(\zeta)}{\tau(\zeta_1)-\zeta_1} - \frac{1-2\sigma}{1-\sigma}\left\{\frac{f^{\bullet}(\zeta_1)}{[\tau(\zeta_1)-\zeta_1)]^2} + \frac{f(\zeta_1)}{[\tau(\zeta_1)-\zeta_1]^3}\right\} = -\Sigma_* - p_h \\
f^{\bullet}(\zeta) + \frac{f(\zeta_1)}{\tau(\zeta)-\zeta_1} = g_1(\zeta_1) + g_2(\zeta_1) + \frac{1-\sigma}{1+\sigma}p_h[\tau(\zeta_1)-\zeta_1]^2 \\
f^{\bullet\bullet}(\zeta_1) + \frac{2(1-2\sigma)}{1-\sigma}\left\{\frac{f^{\bullet}(\zeta_1)}{\tau(\zeta_1)-\zeta_1} + \frac{f(\zeta_1)}{[\tau(\zeta_1)-\zeta_1]^2}\right\} - \lambda^2\left[g_1^{\bullet}(\zeta_1) - \frac{g_2^{\bullet}(\zeta_1)}{(\lambda+1)\tau^{\bullet}(\zeta_1)-1} + \frac{g_1(\zeta_1)+g_2(\zeta_2)}{\tau(\zeta_1)-\zeta_1}\right] + p_h[\tau(\zeta_1) \\
-\zeta_1] = \frac{\tau^{\bullet}(\zeta_1)-1}{\tau^{\bullet}(\zeta_1)}\left\{f^{\bullet\bullet}(\zeta_1) + \frac{f^{\bullet}(\zeta_1)}{\tau(\zeta_1)-\zeta_1} - \lambda\left[g_1^{\bullet}(\zeta_1) + \frac{g_2^{\bullet}(\zeta_1)}{(\lambda+1)\tau^{\bullet}(\zeta_1)-1}\right]\right\}
\end{gathered} \tag{12}$$

where

$$g_2(\zeta_1) = f_2(\eta_1), \quad \frac{g_2^{\bullet}(\zeta_1)}{(\lambda+1)\tau^{\bullet}(\zeta_1)-1} = f_2^{\bullet}(\eta_1) \tag{13}$$

and the functions $g_1(\zeta_1) = f_1(\xi_1)$ and $g_1^{\bullet}(\zeta_1) = f_1^{\bullet}(\xi_1)$ are known. The arguments ξ_1 and η_1 are expressed through ζ_1 by the equations

$$\xi_1 = (\lambda - 1)\tau(\zeta_1) + \zeta_1, \quad \eta_1 = (\lambda + 1)\ \tau(\zeta_1) - \zeta_1 \tag{14}$$

For (12) the Cauchy problem is posed in the interval $\zeta_{11} \le \zeta_1 \le \zeta_{12}$, where $\zeta_{11} = \tau_1 - x_1(\tau)$ and ζ_{12} is found in the process of integrating (12) using the equation

$$\zeta_{12} = \xi_{13} - (\lambda - 1)\ \tau(\zeta_{12}) \tag{15}$$

The initial conditions for (12) with $\zeta_1 = \zeta_{11}$ are taken from the asymptotic solution. Solving the indicated Cauchy problem, we find the function $f(\zeta_1)$ in the interval $[\zeta_{11}, \zeta_{12}]$, $x_1(\tau)$ in $[\tau_1 + \Delta\tau, \tau_2(\zeta_{12})]$ and $f_2(\eta_1)$ in $[\eta_{11}, \eta_{12}]$, where

$$\eta_{11} = \lambda(\tau_1 + \Delta\tau) + 1, \qquad \eta_{12} = (\lambda + 1)\ \tau_2(\zeta_{12}) - \zeta_{12}$$

When converting to the variable $\eta^\circ = \lambda\tau + 1$ the last equation (10) takes the form

$$\lambda^2[l^{\bullet}(\eta^\circ) + f_2^{\bullet}(\eta^\circ) + l(\eta^\circ) + f_2(\eta^\circ)] = p_0\left[1 + l(\eta^\circ) + f_2(\eta^\circ) + \frac{1-\sigma}{1+\sigma}p_h\right]^{-3\gamma} \tag{16}$$

where $l(\eta^\circ) = f(\xi^\circ)$ and $f_2(\eta^\circ)$ is a known function in the interval $\eta_{11} \le \eta^\circ \le \eta_{12}$. Here again the Cauchy problem is posed, for which the initial conditions are again taken for $\eta^\circ = \eta_{11}$ from the asymptotic solution. Solving this problem, we find the function $f_1(\xi_1)$ in the interval $\xi_{12} \le \xi_1 \le \eta_{12} + 2$. We again return to (12) for the new initial conditions corresponding to the value of ζ_{12} from the preceding solution of this same system and for the new interval of variation of the independent variable $\zeta_{12} \le \zeta \le \zeta_{13}$, where ζ_{13} is calculated in the process of constructing the solution using (15) if therein the index 12 is replaced by 13. Thus, after solving this problem we calculate $f(\zeta_1)$, $x_1(\tau)$ and $f_2(\eta_1)$ and $f_2(\eta_1)$ becomes known in the interval $[\eta_{12}, \eta_{13}]$, where $\eta_{13} = (\lambda + 1)\ \tau(\zeta_{13})$, after which we turn to the solution of (16) and so on.

As we have mentioned above, to construct the solution we must have the asymptotic solution near the point $\tau = \tau_1$, $x = 1$, which is constructed by calculating the values of the unknown functions and their derivatives at the point $(\tau_1, 1)$. By virtue of the requirement for continuity of the displacements along the characteristic $\zeta_1 = \tau_1 - 1$ the functions $f(\zeta_1)$ and $f^{\bullet}(\zeta_1)$ must be continuous on this characteristic and, there-

TABLE 1

x	ζ	σ_r	σ_θ	V	x	ζ	σ_r	σ_θ	V
1.0	−1.00	−200.0	−75.0	+6.63	7.0	−0.60	−25.7	−6.3	+0.58
1.0	−0.60	−198.0	−5.2	+5.86	7.0	−0.12	−16.2	−2.4	+0.34
1.0	−0.12	−195.0	+45.0	+4.70	7.3	0.20	−10.6	−0.42	+0.20
1.3	0.20	−130.0*	+45.0	+2.76	7.7	0.80	−4.94	+1.46	+0.06
		(−123.0)	(0)	(+2.15)	7.8	0.80	−2.93	+2.06	+0.002
1.7	0.80	−88.6	+45.0	+1.42	7.9	1.40	−1.38	+2.48	−0.031
		(−85.0)	(0)	(+1.04)	8.0	1.80	+0.04	+2.80	−0.067
1.8	1.10	−76.8	+45.0	+1.02	8.0	2.30	+1.31	+3.00	−0.100
		(274.5)	(0)	(+0.72)	8.0	2.70	+1.90	+2.99	−0.114
1.9	1.40	−68.9	+45.0	+0.72	8.0	3.00	+2.41	+2.99	−0.126
		(−67.3)	(0)	(+0.50)	8.0	3.40	+2.81	+2.90	−0.136
2.0	1.80	−62.3	+45.0	+0.44	8.0	3.80	+3.12	+2.73	−0.141
		(−61.5)	(0)	(+0.29)	15.0	−1.00	−12.7	−5.40	+0.421
2.0	2.30	−57.6	+45.0	+0.16	15.0	−0.60	−7.96	−3.20	+0.261
		(57.3)	(0)	(+0.10)	15.0	−0.12	−4.93	−1.43	+0.144
2.0	2.70	−56.1	+45.0	0.005	15.3	0.20	−2.50	−0.54	0.080
		(−55.8)	(0)	(−0.007)	15.7	0.80	−0.49	+0.35	+0.012
2.0	3.00	−55.0	+42.0	−0.120	15.8	1.10	+0.23	+0.65	−0.012
		(−55.0)	(0)	(−0.12)	15.9	1.40	+0.78	+0.87	−0.030
2.0	3.40	−54.7	+37.0	−0.25	16.0	1.80	+1.28	+1.04	−0.047
		(−54.7)	(0)	(−0.25)	16.0	2.30	+1.69	+1.16	−0.061
2.0	3.80	−55.1	+35.0	−0.39	16.0	2.70	+1.85	+1.18	−0.067
		(−55.1)	(0)	(−0.39)	16.0	3.00	+2.00	+1.20	−0.071
3.0	−1.00	−64.2	−26.0	−0.256	16.0	3.40	+2.07	+1.17	−0.074
3.0	−0.60	−46.3	−12.0	−0.225	16.0	3.80	+2.09	+1.12	−0.074
3.0	−0.12	−32.4	−1.76	−0.140	25.0	−1.00	−7.66	−3.30	+0.253
3.3	0.20	−22.8	+3.18	−0.153	25.0	−0.60	−4.74	−2.00	+0.155
3.7	0.80	−13.4	+7.15	−0.109	25.0	−0.12	−2.52	−0.92	+0.084
3.8	1.10	−10.2	+8.29	−0.029	25.3	0.20	−1.37	−0.40	+0.045
3.9	1.40	−7.72	+9.05	+0.067	25.7	0.80	−0.15	+1.14	+0.004
4.0	1.80	−5.45	+9.60	+0.161	25.8	1.10	+0.29	+0.32	−0.011
4.0	2.30	−3.44	+9.84	+0.283	25.9	1.40	+0.62	+0.46	−0.022
4.0	2.70	−2.48	+9.75	+0.826	26.0	1.80	+0.92	+0.57	−0.032
4.0	3.00	−1.64	+9.65	+0.987	26.0	2.30	+1.16	+0.65	−0.040
4.0	3.40	−0.93	+9.41	+1.493	26.0	2.70	+1.25	+0.67	−0.043
4.0	3.80	−0.32	+8.26	+2.138	26.0	3.00	+1.33	+0.60	−0.046
7.0	−1.00	−38.2	−11.5	+0.91	26.0	3.40	+1.36	+0.69	−0.047

*Numbers in parentheses correspond to the value behind the fracture front.

fore, the functions $f(\tau_1-1) = f_0$ and $f^{\bullet}(\tau_1-1) = f_0^{\bullet}$ will be known. Then from the first equation (10) we find $f_0^{\bullet\bullet}$.* Since only the sum $f_1(\xi) + f_2(\eta)$ appears in (9), one of the terms in the sum $f_{10} + f_{20}$ can be specified arbitrarily, for example we can set $f_{20} = 0$.

From (10) for $\tau = \tau_1$, x = 1 it is easy to calculate $f_0^{\bullet\bullet}$, f_{10}, $f_{10}^{\bullet}$, $f_{20}^{\bullet}$, and $x_{10}^{\bullet} = 0$ [1]. The functions $x_1^{\bullet\bullet}(\tau)$, $f_1^{\bullet\bullet}(\xi_1)$, $f_2^{\bullet\bullet}(\eta_1)$, $f^{(IV)}(\zeta_1)$ have singularities of order $x_1^{\bullet\bullet} \sim (\tau-\tau_1)^{-1/2}$, $f_1^{\bullet\bullet} \sim (\xi_1-\xi_{10})^{-1/2}$, $f_2^{\bullet\bullet} \sim (\eta_1-\eta_{10})^{-1/2}$, $f^{(IV)} \sim (\zeta_1-\zeta_{10})^{-1/2}$ for $\tau \to \tau_1$, $x \to 1$. With account for the above, we obtain the asymptotic behavior of the solution of (10) near $\tau = \tau_1$, x = 1 in the form

$$
\begin{gathered}
x_1(\tau) = 1 + a(\tau-\tau_1)^{3/2} + \ldots \\
f(\zeta_1) = f_0 + f_0^{\bullet}(\tau-\tau_1) - 1.5af_0^{\bullet}(\tau-\tau_1)^{3/2} + 0.5f_0^{\bullet\bullet}(\tau-\tau_1)^2 \\
- af_0^{\bullet\bullet}(\tau-\tau_1)^{5/2} + 0.5(0.33f_0^{\bullet\bullet\bullet} + a^2 f_0^{\bullet\bullet})(\tau-\tau_1)^3 + \ldots \\
f_1(\xi_1) = f_{10} + \lambda f_{10}^{\bullet}(\tau-\tau_1) + (1{,}33\lambda^2 b - f_{10}^{\bullet} a)(\tau-\tau_1)^{3/2} \\
+ 0.5[\lambda c + 1.5ab(1+2\lambda)](\tau-\tau_1)^2 + 0.2[(2-3\lambda)ac + 1.8a^2 b](\tau-\tau_1)^{5/2} + \ldots \\
f_2(\eta_1) = \lambda f_{20}^{\bullet}(\tau-\tau_1) + (1.33b\lambda^2 + af_{20}^{\bullet})(\tau-\tau_1)^{3/2} \\
+ 0.75(1-2\lambda)ab(\tau-\tau_1)^2 + 0.9a^2 b(\tau-\tau_1)^{5/2} + \ldots
\end{gathered}
\tag{17}
$$

where

$$
a = 0.66\left\{\left[2f_0^{\bullet\bullet\bullet} + 2\left(\frac{2(1-2\sigma)}{1-\sigma} + \vartheta\right)\right]\frac{f_2^{\bullet\bullet} + f_0^{\bullet}}{\theta}\right\}^{1/2}
$$

$$
b = \frac{a\theta}{4\lambda^2}, \quad c = -\frac{\lambda^2 + \vartheta}{\lambda(f_0^{\bullet\bullet} + f_0^{\bullet})}
$$

* Hereafter functions with subscript 0 correspond to the value of this function at the point $\tau = \tau_1$, x = 1.

$$
\begin{aligned}
\theta &= (1-\lambda^2) f_0^{\bullet\bullet\bullet} + \frac{3+\sigma}{1-\sigma} f_0^{\bullet} + \frac{2}{1-\sigma} f_0 + \frac{2(1-\sigma)}{\lambda^2} p_n - f_{10} \\
\vartheta &= 3p_0\gamma \left[1 + f_{10} + \frac{1-\sigma}{1+\sigma} p_h\right]^{-1-3\gamma} \\
f_0^{\bullet\bullet} &= \frac{1-2\sigma}{\sigma}(f_0^{\bullet} + f_0) - \frac{1-\sigma}{\sigma}(\Sigma_* + p_h) \\
f_0^{\bullet\bullet\bullet} &= \frac{1-2\sigma}{\sigma}(f_0^{\bullet\bullet} + f_0^{\bullet}), \quad f_{10} = f_0^{\bullet} + f_0 + \frac{1-\sigma}{1+\sigma} p_h \\
f_{10}^{\bullet} &= \frac{\lambda^2}{2} f_0^{\bullet\bullet} + \frac{1}{1-\sigma}(f_0^{\bullet} + f_0) + \frac{1}{2\lambda^2} p_h - 0.5 f_{10} + \frac{1}{2\lambda}(f_0^{\bullet\bullet} + f_0^{\bullet}) \\
f_{20}^{\bullet} &= \frac{1}{2\lambda}(f_0^{\bullet\bullet} + f_0^{\bullet}) - \frac{\lambda^2}{2} f_0^{\bullet\bullet} - \frac{1}{1-\sigma}(f_0^{\bullet} + f_0) - \frac{1}{2\lambda^2} p_h + 0.5 f_{10}
\end{aligned}
\tag{18}
$$

A unified program was written for the Strela-4 computer of the Moscow University Computing Center which permits calculating the problem solution for all the stages using the scheme described above. This program was used to calculate several versions for different media and for different P_0 and P_h.

Figure 2 shows calculation results which are typical for all the cases. The lines 1 and 3 show the nature of the fracture front movement in granite $P_0 = 500$ at, $P_h = 100$ at and $P_0 = 1500$ at, $P_h = 100$ at respectively, the lines 2 and 4 are for shales ($\sigma = 0.26$, $E = 1.9 \cdot 10^5$ kg/cm^2, $\sigma_* = 38$ kg/cm^2 [2]) with $P_0 = 500$ at, $P_h = 100$ at and for limestone ($\sigma = 0.25$, $E = 7 \cdot 10^5$ kg/cm^2, $\sigma_* = 25.5$ kg/cm^2 [2]) for $P_0 = 1000$ at, $P_h = 100$ at, respectively. The data of Fig. 2 show that all the qualitatively different characteristics in the fracture front movement zone noted above are actually observed, depending on the problem conditions. In the case corresponding to curves 1, 2 (relatively low pressures in the cavity), the crack front begins to displace back in the direction of the cavity after σ_θ decreases to zero and oscillations take place in this process. In the case corresponding to curve 3 (relatively high pressure in the cavity) this reverse movement of the crack front and its oscillations are not observed. Finally, in the case corresponding to curve 4 (relatively low tearing strength $\sigma_* = 25.5$ kg/cm^2), new fracturing develops after the first stopping of the fracture front and its subsequent reverse movement with oscillations. This last effect is associated with the fact that, as shown by the calculation, the value of σ_θ on the front side of the fracture front after the moment it stops does not change monotonically, but performs oscillations, therefore for a small value of σ_* these oscillations of σ_θ lead to repeated reaching of the values σ_* and to the development of repeat fractures. The calculations for the other versions showed that repeated fracture can occur a large number of times.

Figure 1b shows the maximal values of the fracture zone radius in granite as a function of P_0 and P_h analogous to Fig. 1a (the isolines are for $x_1 = x_1(P_0, P_h)$).

Figure 3 shows the nature of the fracture front propagation in time for different values of σ_* in kg/cm^2 in a medium with the mechanical characteristics $\sigma = 0.3$, $E = 2.22 \cdot 10^5$ kg/cm^2 for an explosion with parameters $P_0 = 200$ at, $P_h = 0$. We see that with decrease of σ_* the fracture zone enlarges and for $\sigma_* = 0$ the fracture front velocity becomes constant and equal to C_1. The exact solutions of the given problem can be constructed for this case [3].

Table 1 shows the values of the stresses σ_r, σ_θ in kg/cm^2 and the mass velocities V as a function of $\xi = \tau - x$ and x from the results of the calculation for $P_0 = 200$ at, $P_h = 0$ in granite. In Figs. 4 and 5 the solid curves show the stresses, mass velocities, and displacements at the distance $x = 100$ from the center of the explosion in the fracture case; the dashed curves are without fracture for an explosion in granite with the parameters $P_0 = 200$ at, $P_h = 0$. In these figures the section 1-1 corresponds to the moment the elastic disturbance, radiated at the moment of fracture at the boundary of the cavity, reaches the section $x = 100$. We see that account for fracture alters qualitatively the form of the elastic wave radiated from the center of the explosion, although there is no significant change of the amplitude and duration of this wave. However, for a more intense explosion (higher pressures P_0), when in the initial stage the medium around the cavity is involved in the plastic flow and a strong compression shock wave propagates in this region, the elastic wave at large distances from the explosion source is changed both qualitatively and quantitatively – account for these effects leads to more intense decay of the wave amplitude with distance and a significant (by an order of magnitude) increase of its duration [4].

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